FLORENCE NIGHTINGALE DAY 12TH JANUARY 2017

QUIZ – SOLUTIONS

Q1.

Alex, Ben, Charles and Dave want to cross a bridge. It is dark, and they have to light the path with a torch. No more than two people can cross the bridge together, and the group has only one torch. Alex crosses the bridge in 1 minute, Ben crosses the bridge in 2 minutes, Charles crosses the bridge in 5 minutes, Dave crosses the bridge in 10 minutes. How can the group cross the bridge in 17 minutes?

Solution. Alex and Ben cross first (2'). Ben returns (4'). Charles and Dave cross (14'). Alex returns (15'). Alex and Ben cross (17') and all four have crossed the bridge in 17'. (Equivalently, Alex returns first instead of Ben, who returns after Charles and Dave have crossed the bridge.)

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Q2.

The frame of a square painting is 30cm wide. The area of the painting is equal to that of the frame. What is the length of a side of the painting?

Solution.

Let x be the length of a side of the painting. Then the area of the painting is x^2 and that of the frame is $2\big((30)(60+x)+30x\big) = 120(x+30)$. So

$$x^2 - 120x - 3600 = 0 \quad \text{giving} \quad x = 60(1+\sqrt{2}) \approx 150\text{cm}$$

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Q3.

You have three bags, each containing two marbles. Bag A contains two white marbles, Bag B contains two black marbles, and Bag C contains one white marble and one black marble.

You pick a random bag and take out one marble.

It is a white marble.

What is the probability that the remaining marble from the same bag is also white?

Solution.

The probability is $\frac{2}{3}$. Indeed, you know that you do not have Bag B (two black marbles) so there are three possibilities:

You chose Bag A, first white marble. The other marble will be white

You chose Bag A, second white marble. The other marble will be white

You chose Bag C, the white marble. The other marble will be black

So there are 2 out of 3 possibilities for a white marble

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Q4. *Covent Garden Problem, due to Samuel Loyd, 1841–1911*

Mrs. Smith and Mrs. Jones had equal number of apples but Mrs. Jones had larger fruits and was selling hers at the rate of two for a penny, while Mrs. Smith sold three of hers for a penny.

Mrs. Smith was for some reason called away and asked Mrs. Jones to dispose of her stock. Upon accepting the responsibility of disposing her friend's stock, Mrs. Jones mixed them together and sold them off at the rate of five apples for two pence.

When Mrs. Smith returned the next day the apples had all been disposed of, but when they came to divide the proceeds they found that they were just seven pence short.

Supposing that they divided the money equally, each taking one-half, how much money did Mrs. Jones lose in this partnership?

Solution.

The mixed apples were sold off at the rate of five apples for two pence. So they must each have had a multiple of five apples, and also a multiple of 2 (Mrs. Jones) or 3 (Mrs. Smith) apples.

So the minimum number of apples they could have together is 60. Then, 30 would have been of Mrs. Smith's that would fetch her 10 (an integer) pence and the other 30 of Mrs. Jones's that would fetch her 15 (also an integer) pence.

When sold separately it would fetch them 10+15=25 pence altogether. But when sold together it would fetch them $\dfrac{60 \times 2}{5} = 24$ pence i.e. a loss of one (25-24=1) penny.

Since they lost 7 pence altogether; they had altogether $60 \times 7 = 420$ apples that fetched them only $\dfrac{420 \times 2}{5} = 168$ pence and they shared 84 pence each of them. But Mrs. Jones could sell her $\dfrac{420}{2} = 210$ apples for $\dfrac{210}{2} = 105$ pence so she lost **21 pence**.

Note that Mrs. Jones lost 21 pence. But without working Mrs. Smith earned 14 extra pence!

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Q5.

If we wrote down all numbers of 5 digits with all even digits, not starting with 0, and without repetitions of digits in a number, and listed them in increasing order, which would be the 50th number?

Solution.

60284. Indeed, choosing the first digit gives $4! = 24$ numbers starting with that digit. Since $50 = 2 \cdot 24 + 2$, the 50th number is the second (in the increasing order) starting with the third possible starting digit, namely 6. The result follows.

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Q6.

A bicycle rider went a mile in three minutes with the wind, and returned in four minutes against the wind. How fast could he ride a mile if there was no wind?

Solution.

Note that the wind has helped the rider for only three minutes, while it has worked adversely for four minutes.

If the rider could ride a mile in three minutes with the wind, then he could go a mile and a third in four minutes, and one mile in four minutes against the wind.

Therefore $1 + 1\frac{1}{3} = \frac{7}{3}$ miles in eight minutes gives his actual speed, because the wind helped him just as much as it has retarded him.

So his actual speed for a single mile without any wind is $\frac{8 \cdot 3}{7} = \frac{24}{7} = 3\frac{3}{7}$ minutes (i.e. 3 minutes and 25.7 seconds approx.)

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Q7.

Andy has written a sequence of consecutive integers, starting from 1, on a blackboard. Bill comes after Andy has left and erases one of those in the list. Cathy comes after Bill has left and she calculates that the average of the numbers left on the board is $35 + \frac{7}{17}$. Which number has been erased?

Solution.

Since the numbers are integers, the average suggests that the list should have comprised n numbers with $n \equiv 1 \pmod{17}$. We can check by hand, starting with $n = 18$ if we could get such an average. The first possibility we obtain is that Andy wrote the integers $1, \ldots, 69$ and Bill has erased the number 7.

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Q8.

Find a 5 digits integer such that if we add a 1 in front (e.g. 33333 becomes 133333), then we get a number equal to a third to the number formed by the same 5 digits followed by a 1 (e.g. 33333 becomes 333331). Find the 5 digits number.

Solution.

Let x be the 5 digit number. Then

$$3(100000 + x) = 10x + 1 \quad \text{i.e.} \quad 299000 = 7x$$

since adding 100000 puts a 1 in front of x, and multiplying by 10 and adding 1 puts a 1 at the end of x.

We thus obtain $x = \frac{299999}{7} = 42857 \quad (\text{check: } 3 \times 142857 = 428571)$

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Q9.

Lizzie has drawn 100 segments of 1cm so as to obtain the maximum possible of equilateral triangles. How many triangles has she obtained?

Solution.

60, as shown in Figure 1: draw an hexagon with sides of 3 cm. Filling it as a complete lattice requires $3(3+4+5+6+5+4+3) = 90$ segments of 1 cm, and we obtain $2(3+4+4+5+5+6) = 54$ triangles of 1cm side length. We are left with 10 segments to draw, and we can draw 6 more triangles. Hence a total of 60.

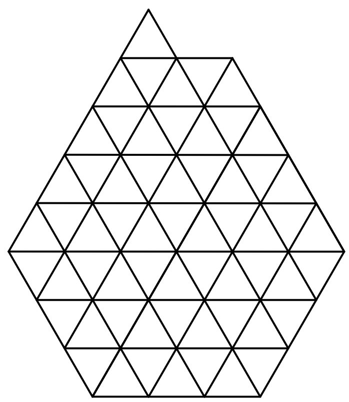

FIGURE 1. 100 segments of 1cm make up 60 equilateral triangles

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Q10.

Five friends have their gardens next to one another, where they grow three kinds of crops: fruits (apple, pear, nut, cherry), vegetables (carrot, parsley, gourd, onion) and flowers (aster, rose, tulip, lily).

1. The 12 different varieties are indeed cultivated. 2. Everybody grows exactly 4 different varieties. 3. Each variety is at least in one garden. 4. Only one variety is in 4 gardens. 5. Only in one garden are all 3 kinds of crops. 6. Only in one garden are all 4 varieties of one kind of crops. 7. Pear is only in the two border gardens. 8. Paul's garden is in the middle with no lily. 9. Aster grower doesn't grow vegetables. 10. Rose growers don't grow parsley. 11. Nuts grower has also gourd and parsley. 12. In the first garden are apples and cherries. 13. Only in two gardens are cherries. 14. Sam has onions and cherries. 15. Luke grows exactly two kinds of fruit. 16. Tulip is only in two gardens. 17. Apple is in a single garden. 18. Only in one garden next to Zick's is parsley. 19. Sam's garden is not on the border. 20. Hank grows neither vegetables nor asters. 21. Paul has exactly three kinds of vegetable.

Who has which garden and what is grown where?

Solution.

Hank: pear apple cherry rose // Sam: cherry onion rose tulip // Paul: carrot gourd onion rose // Zick: aster rose tulip lily // Luke: pear nut gourd parsley