Methods for Univariate Extreme Value Inference

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Introduction

Figure: Fort Collins, Colorado, USA

Introduction

Figure: Fort Collins Flood, July 1997

Outline

Peaks Over Threshold

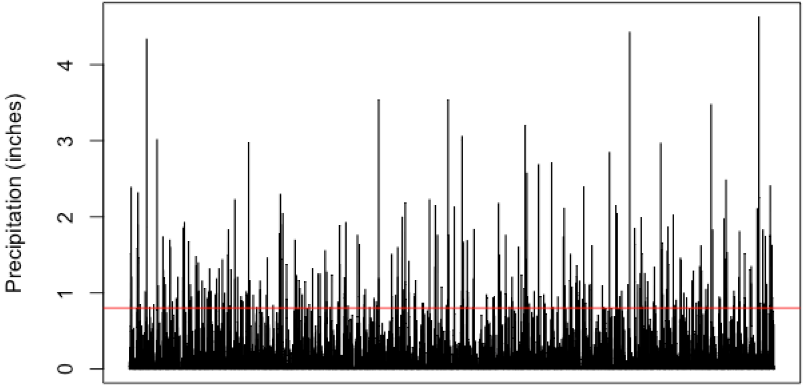


Figure: Fort Collins Daily Maximum Precipitation Data with Threshold, 1900-1999

Density function of **generalised Pareto distribution** (GPD) is

$$g_{\xi,\sigma}(x) = \begin{cases} \frac{1}{\sigma}\left(1 + \frac{\xi x}{\sigma}\right)^{-1-1/\xi} & \text{for } \xi \neq 0 \\ \frac{1}{\sigma}\, e^{-x/\sigma} & \text{for } \xi = 0, \end{cases}$$

where $x > 0$.

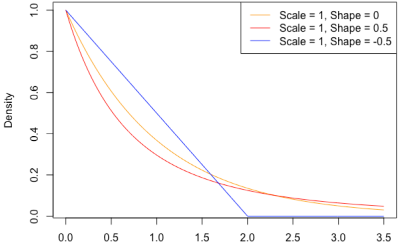


Figure: GPD Density Functions with Different Shape Parameters

Method of Murphy *et al.*

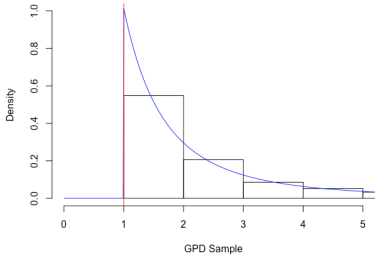
Input: data, proposal thresholds, # of bootstraps = k, # of quantile levels = m.

For each of the proposal thresholds:

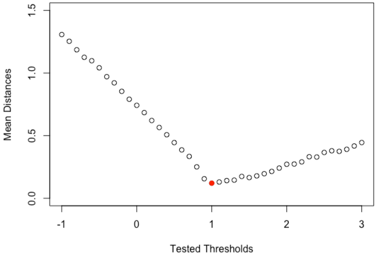
1. Estimate the GPD parameters using MLE.
2. Measure average distance at m equally spaced quantile levels between estimated GPD and data exceedance.
3. Bootstrap the exceedances over the proposed threshold and repeat Step 2 for k times.
4. Compute mean of the k average distances.

Output: proposal threshold with least mean average distance.

Method of Murphy *et al.*



(a) Method of Murphy *et al.* with GPD Samples



(b) Mean Distance Plot with Minimum in Red

Mixture Model

- Model the extreme by one distribution (e.g. GPD)
- Model the non-extreme by another distribution (e.g. Gaussian)
- Average the two distributions by imposing a (continuous / discrete) weight.

Dynamically Weight Model (DWM)

Based on Frigessi, Haug, and Rue (2002).

Dynamically Weight Model has the density function

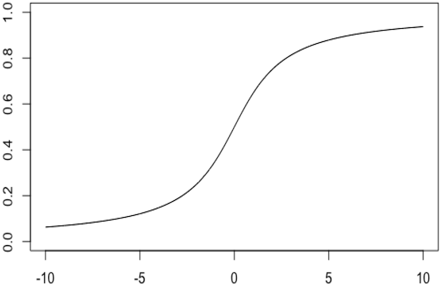
$$l(x) = \frac{(1 - p(x))f(x) + p(x)g(x)}{Z}$$

where

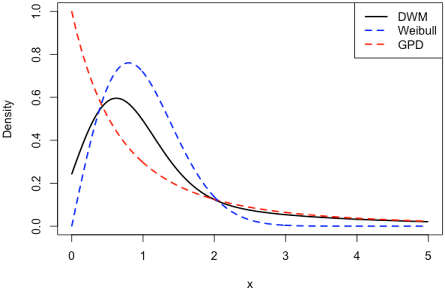
$$\begin{aligned} g(x) &:= \frac{1}{\sigma}\left(1 + \frac{\xi x}{\sigma}\right)^{-1-1/\xi}, \\ f(x) &:= \beta\lambda^{\beta}x^{\beta-1}\exp[-(\lambda x)^{\beta}], \\ p(x) &:= \frac{1}{2} + \frac{1}{\pi}\arctan\left(\frac{x-\mu}{\tau}\right). \end{aligned}$$

g is GPD density, f is Weibull density, and p is Cauchy CDF. Z is a normalising constant.

Dynamically Weight Model (DWM)



(a) Weight Function with $\mu = 0$ and $\tau = 2$



(b) Dynamically Weighted Mixture Model

Hybrid Pareto Model Distribution (HPD)

Based on Carreau and Bengio (2009).

Hybrid Pareto Model Distribution has the density function

$$h(y) = \begin{cases} \frac{1}{\gamma} f_{\mu,\sigma_N}(y) & y \leq u \\ \frac{1}{\gamma} g_{\xi,\sigma}(y-u) & y > u, \end{cases}$$

where f_{μ,σ_N} is the normal density, $g_{\xi,\sigma}$ is the GPD density, and γ is the normalising constant. u is the threshold.

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To enforce continuity of the distribution, two constraints are imposed at the threshold u:

(1) continuity at u, $f(u) = g(0)$

(2) continuity of derivative at u, $f'(u) = g'(0)$

Hybrid Pareto Model Distribution (HPD)

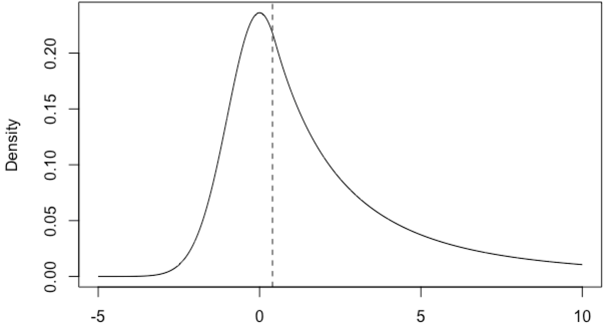


Figure: Hybrid Pareto Distribution with $\mu = 0, \sigma = 1, \xi = 0.4$

Simulation Study - Setup

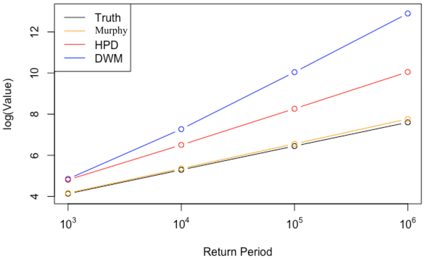
Models: (1) Method of Murphy *et al.* (2) HPD (3) DWM

Sample Distributions: (1) GPD scale = 1, shape = 0.5, threshold = 1 (2) Standard Normal (3) Beta (2,5)

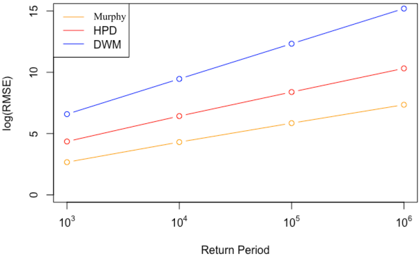
Sample Size = 1000

Repetition = 100.

Comparisons: (1) 10^3, 10^4, 10^5, 10^6 Return Levels. (2) Root Mean Squared Errors

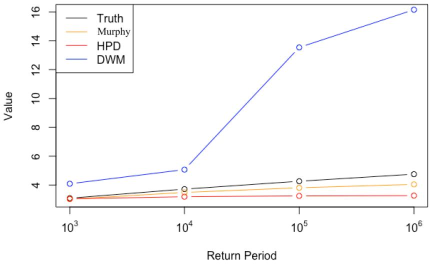


(a) Estimated Return Levels

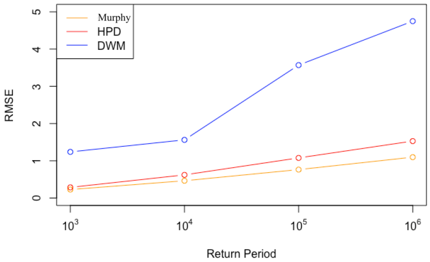


(b) Root Mean Squared Errors

Simulation Study - Standard Normal Samples

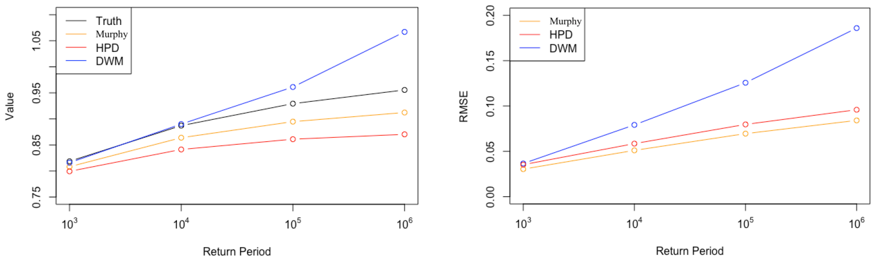


(a) Estimated Return Levels



(b) Root Mean Squared Errors

Simulation Study - Beta(2,5) Samples



(a) Estimated Return Levels (b) Root Mean Squared Errors

Conclusion

Summary

- Method of Murphy *et al.* provides most accurate return level estimations and lowest RMSEs.
- HPD provides relatively accurate return level estimations too.

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Extensions

- Simulation studies for an even more varied sample distributions.
- Incorporate confidence intervals for estimations by Murphy *et al.*

Questions?